

Exercise 22

Solve the *initial-value problem* (Debnath 1994, p. 115) for the two-dimensional surface waves at the free surface of a running stream of velocity U . The problem satisfies the following equation, boundary, and initial conditions:

$$\begin{aligned} \phi_{xx} + \phi_{zz} &= 0, & -\infty < x < \infty, & -h \leq z \leq 0, & t > 0, \\ \left. \begin{aligned} \phi_x + U\phi_x + g\eta &= -\frac{P}{\rho}\delta(x)\exp(i\omega t), \\ \eta_t + U\eta_x - \phi_z &= 0 \end{aligned} \right\} & \text{on } z = 0, & t > 0, \\ \phi(x, z, 0) = \eta(x, 0) &= 0, & \text{for all } x \text{ and } z. \end{aligned}$$

[TYPO: This should be ϕ_t !]