Exercise 28

Use the Fourier transform to solve the Rossby wave problem in an inviscid β-plane ocean bounded by walls at $y = 0$ and $y = 1$, where y and x represent vertical and horizontal directions. The fluid is initially at rest and then, at $t = 0+$, an arbitrary disturbance localized to the vicinity of $x = 0$ is applied to generate Rossby waves. This problem satisfies the Rossby wave equation

$$\frac{\partial}{\partial t}[(\nabla^2 - \kappa^2)\psi] + \beta\psi_x = 0, \quad -\infty < x < \infty,\ 0 \le y \le 1,\ t > 0,$$

with the boundary and initial conditions

$$\psi_x(x, y) = 0 \quad \text{for } 0 < x < \infty,\ y = 0 \text{ and } y = 1,$$
$$\psi(x, y, t) = \psi_0(x, y) \quad \text{at } t = 0 \text{ for all } x \text{ and } y.$$

Examine the case for $\psi_{0n}(x) = \frac{1}{\alpha\sqrt{2}} \exp\{ik_0 x - (\frac{x}{a})^2\}$.