Exercise 29

The equations for the current $I(x,t)$ and potential $V(x,t)$ at a point x and time t of a transmission line containing resistance R, inductance L, capacitance C, and leakage inductance G are

$$LI_t + RI = -V_x, \quad \text{and} \quad CV_t + GV = -I_x.$$

Show that both I and V satisfy the telegraph equation

$$\frac{1}{c^2}u_{tt} - u_{xx} + au_t + bu = 0,$$

where $c^2 = (LC)^{-1}$, $a = LG + RC$, and $b = RG$.
Solve the telegraph equation for the following cases with $R = 0$ and $G = 0$:

(a) $V(x,t) = V_0 H(t)$ at $x = 0,\ t > 0,\ V(x,t) \to 0$ as $x \to \infty,\ t > 0$, where V_0 is constant.

(b) $V(x,t) = V_0 \cos \omega t$ at $x = 0,\ t > 0,\ V(x,t) \to 0$ as $x \to \infty,\ t > 0$.