

Exercise 34

Solve the Stokes problem which is concerned with the unsteady boundary layer flows induced in a semi-infinite viscous fluid bounded by an infinite horizontal disk at $z = 0$ due to nontorsional oscillations of the disk in its own plane with a given frequency ω . The equation of motion and the boundary and initial conditions are

$$\begin{aligned}u_t &= \nu u_{zz}, & z > 0, t > 0, \\u(z, t) &= U e^{i\omega t} & \text{on } z = 0, t > 0, \\u(z, t) &\rightarrow 0 & \text{as } z \rightarrow \infty \text{ for } t > 0, \\u(z, 0) &= 0 & \text{for } t \leq 0 \text{ and } z > 0,\end{aligned}$$

where $u(z, t)$ is the velocity of the fluid of kinematic viscosity ν and U is constant. Solve the Rayleigh problem ($\omega = 0$). Explain the physical significance of both the Stokes and Rayleigh solutions. [TYPO: This should be t .]