

Exercise 39

Use the Laplace transform to solve the initial boundary-value problem

$$\begin{aligned}u_t &= c^2 u_{xx}, & 0 < x < a, t > 0, \\u(x, 0) &= x + \sin\left(\frac{3\pi x}{a}\right) & \text{for } 0 < x < a, \\u(0, t) &= 0 = u(a, t) & \text{for } t > 0.\end{aligned}$$