

Exercise 4

- (a) Show that the nonlinear equation

$$u^2 u_{xx} + 2u_x u_y u_{xy} - u^2 u_{yy} = 0$$

is hyperbolic for every solution $u(x, y)$.

- (b) Show that the nonlinear equation for the velocity potential $u(x, y)$

$$(1 - u_x^2)u_{xx} - 2u_x u_y u_{xy} + (1 - u_y^2)u_{yy} = 0$$

in certain kinds of compressible fluid flow is (i) elliptic, (ii) parabolic, or (iii) hyperbolic for those solutions such that $|\nabla u| < 1$, $|\nabla u| = 1$, or $|\nabla u| > 1$.