

Exercise 41

Use the joint Laplace and Fourier transform to solve the initial-value problem for transient water waves which satisfies (see Debnath 1994, p. 92)

$$\begin{aligned} \nabla^2 \phi &= \phi_{xx} + \phi_{zz} = 0, & -\infty < x < \infty, & -\infty < z < 0, & t > 0, \\ \left. \begin{aligned} \phi_z &= \eta_t, \\ \phi_t + g\eta &= -\frac{P}{\rho} p(x) e^{i\omega t} \end{aligned} \right\} & \text{on } z = 0, & t > 0, \\ \phi(x, z, 0) &= 0 = \eta(x, 0) & \text{for all } x \text{ and } z, \end{aligned}$$

where P and ρ are constants.