

Exercise 42

Show that the solution of the boundary-value problem

$$u_{rr} + \frac{1}{r}u_r + u_{zz} = 0, \quad 0 < r < \infty, \quad 0 < z < \infty,$$
$$u(r, z) = \frac{1}{\sqrt{a^2 + r^2}} \quad \text{on } z = 0, \quad 0 < r < \infty,$$

is

$$u(r, z) = \int_0^\infty e^{-\kappa(z+a)} J_0(\kappa r) d\kappa = \frac{1}{\sqrt{(z+a)^2 + r^2}}.$$