Exercise 43

(a) The axisymmetric initial-value problem is governed by

$$u_t = \kappa\left(u_{rr} + \frac{1}{r}u_r\right) + \delta(t)f(r), \quad 0 < r < \infty,\ t > 0,$$
$$u(r,0) = 0 \quad \text{for } 0 < r < \infty.$$

Show that the formal solution of this problem is

$$u(r,t) = \int_0^\infty kJ_0(kr)\tilde{f}(k)\exp(-k^2\kappa t)\,dk.$$

(b) When $f(r) = \frac{Q}{\pi a^2}H(a-r)$, show that the solution is

$$u(r,t) = \frac{Q}{\pi a}\int_0^\infty J_0(kr)J_1(ak)\exp(-k^2\kappa t)\,dk.$$