Bonus Identity

$$\nabla\left[\mathbf{A}\cdot(\mathbf{B}\times\mathbf{r})\right]=\mathbf{A}\times\mathbf{B}$$

A and **B** are constant vectors, and **r** is the position vector.

Proof

$$\begin{aligned}
\nabla\left[\mathbf{A}\cdot(\mathbf{B}\times\mathbf{r})\right] &= \sum_{i=1}^{3}\boldsymbol{\delta}_i\frac{\partial}{\partial x_i}\left\{\left(\sum_{j=1}^{3}\boldsymbol{\delta}_jA_j\right)\cdot\left[\left(\sum_{k=1}^{3}\boldsymbol{\delta}_kB_k\right)\times\left(\sum_{l=1}^{3}\boldsymbol{\delta}_lx_l\right)\right]\right\}\\
&= \sum_{i=1}^{3}\boldsymbol{\delta}_i\frac{\partial}{\partial x_i}\left\{\left(\sum_{j=1}^{3}\boldsymbol{\delta}_jA_j\right)\cdot\left[\sum_{k=1}^{3}\sum_{l=1}^{3}(\boldsymbol{\delta}_k\times\boldsymbol{\delta}_l)B_kx_l\right]\right\}\\
&= \sum_{i=1}^{3}\boldsymbol{\delta}_i\frac{\partial}{\partial x_i}\left[\left(\sum_{j=1}^{3}\boldsymbol{\delta}_jA_j\right)\cdot\left(\sum_{k=1}^{3}\sum_{l=1}^{3}\sum_{m=1}^{3}\boldsymbol{\delta}_m\varepsilon_{klm}B_kx_l\right)\right]\\
&= \sum_{i=1}^{3}\boldsymbol{\delta}_i\frac{\partial}{\partial x_i}\left[\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3}\sum_{m=1}^{3}(\boldsymbol{\delta}_j\cdot\boldsymbol{\delta}_m)\varepsilon_{klm}A_jB_kx_l\right]\\
&= \sum_{i=1}^{3}\boldsymbol{\delta}_i\frac{\partial}{\partial x_i}\left(\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3}\sum_{m=1}^{3}\delta_{jm}\varepsilon_{klm}A_jB_kx_l\right)\\
&= \sum_{i=1}^{3}\boldsymbol{\delta}_i\frac{\partial}{\partial x_i}\left(\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3}\varepsilon_{klj}A_jB_kx_l\right)\\
&= \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3}\boldsymbol{\delta}_i\varepsilon_{klj}\frac{\partial}{\partial x_i}A_jB_kx_l\\
&= \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3}\boldsymbol{\delta}_i\varepsilon_{klj}A_jB_k\frac{\partial x_l}{\partial x_i}\\
&= \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3}\boldsymbol{\delta}_i\varepsilon_{klj}A_jB_k\delta_{il}\\
&= \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\boldsymbol{\delta}_i\varepsilon_{kij}A_jB_k\\
&= \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\boldsymbol{\delta}_i\varepsilon_{jki}A_jB_k\\
&= \sum_{j=1}^{3}\sum_{k=1}^{3}(\boldsymbol{\delta}_j\times\boldsymbol{\delta}_k)A_jB_k\\
&= \left(\sum_{j=1}^{3}\boldsymbol{\delta}_jA_j\right)\times\left(\sum_{k=1}^{3}\boldsymbol{\delta}_kB_k\right)=\mathbf{A}\times\mathbf{B}
\end{aligned}$$