

**Problem 2.22**

Evaluate the following integrals:

(a)  $\int_{-3}^{+1} (x^3 - 3x^2 + 2x - 1) \delta(x + 2) dx.$

(b)  $\int_0^{\infty} [\cos(3x) + 2] \delta(x - \pi) dx.$

(c)  $\int_{-1}^{+1} \exp(|x| + 3) \delta(x - 2) dx.$

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**Solution****Part (a)**

Set the argument of the delta function equal to zero and solve for  $x$ .

$$x + 2 = 0$$

$$x = -2$$

Since  $x = -2$  lies in the interval between  $-3$  and  $+1$ , the delta function sifts out the value of the function next to it at  $x = -2$ .

$$\int_{-3}^{+1} (x^3 - 3x^2 + 2x - 1) \delta(x + 2) dx = (-2)^3 - 3(-2)^2 + 2(-2) - 1 = -25$$

**Part (b)**

Set the argument of the delta function equal to zero and solve for  $x$ .

$$x - \pi = 0$$

$$x = \pi$$

Since  $x = \pi$  lies in the interval between  $0$  and  $\infty$ , the delta function sifts out the value of the function next to it at  $x = \pi$ .

$$\int_0^{\infty} [\cos(3x) + 2] \delta(x - \pi) dx = \cos[3(\pi)] + 2 = 1$$

**Part (c)**

Set the argument of the delta function equal to zero and solve for  $x$ .

$$x - 2 = 0$$

$$x = 2$$

Since  $x = 2$  lies outside the interval between  $-1$  and  $+1$ , the delta function is zero.

$$\int_{-1}^{+1} \exp(|x| + 3) \delta(x - 2) dx = 0$$