

Exercise 1.2.9

Consider a thin one-dimensional rod without sources of thermal energy whose lateral surface area is not insulated.

- (a) Assume that the heat energy flowing out of the lateral sides per unit surface area per unit time is $w(x, t)$. Derive the partial differential equation for the temperature $u(x, t)$.
- (b) Assume that $w(x, t)$ is proportional to the temperature difference between the rod $u(x, t)$ and a known outside temperature $\gamma(x, t)$. Derive

$$c\rho\frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left(K_0 \frac{\partial u}{\partial x} \right) - \frac{P}{A}[u(x, t) - \gamma(x, t)]h(x), \quad (1.2.15)$$

where $h(x)$ is a positive x -dependent proportionality, P is the lateral perimeter, and A is the cross-sectional area.

- (c) Compare (1.2.15) with the equation for a one-dimensional rod whose lateral surfaces are insulated, but with heat sources.
- (d) Specialize (1.2.15) to a rod of circular cross section with constant thermal properties and 0° outside temperature.
- (e) Consider the assumptions in part (d). Suppose that the temperature in the rod is uniform [i.e., $u(x, t) = u(t)$]. Determine $u(t)$ if initially $u(0) = u_0$.