

Exercise 1.4.11

Suppose $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + x$, $u(x, 0) = f(x)$, $\frac{\partial u}{\partial x}(0, t) = \beta$, $\frac{\partial u}{\partial x}(L, t) = 7$.

- (a) Calculate the total thermal energy in the one-dimensional rod (as a function of time).
- (b) From part (a), determine a value of β for which an equilibrium exists. For this value of β , determine $\lim_{t \rightarrow \infty} u(x, t)$.