

Exercise 1.4.7

For the following problems, determine an equilibrium temperature distribution (if one exists). For what values of β are there solutions? Explain physically.

- (a) $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 1, \quad u(x, 0) = f(x), \quad \frac{\partial u}{\partial x}(0, t) = 1, \quad \frac{\partial u}{\partial x}(L, t) = \beta$
- (b) $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad u(x, 0) = f(x), \quad \frac{\partial u}{\partial x}(0, t) = 1, \quad \frac{\partial u}{\partial x}(L, t) = \beta$
- (c) $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + x - \beta, \quad u(x, 0) = f(x), \quad \frac{\partial u}{\partial x}(0, t) = 0, \quad \frac{\partial u}{\partial x}(L, t) = 0$