

Exercise 2.3.2

Consider the differential equation

$$\frac{d^2\phi}{dx^2} + \lambda\phi = 0.$$

Determine the eigenvalues λ (and corresponding eigenfunctions) if ϕ satisfies the following boundary conditions. Analyze three cases ($\lambda > 0$, $\lambda = 0$, $\lambda < 0$). You may assume that the eigenvalues are real.

- (a) $\phi(0) = 0$ and $\phi(\pi) = 0$
- (b) $\phi(0) = 0$ and $\phi(1) = 0$
- (c) $\frac{d\phi}{dx}(0) = 0$ and $\frac{d\phi}{dx}(L) = 0$ (If necessary, see Section 2.4.1.)
- (d) $\phi(0) = 0$ and $\frac{d\phi}{dx}(L) = 0$
- (e) $\frac{d\phi}{dx}(0) = 0$ and $\phi(L) = 0$
- (f) $\phi(a) = 0$ and $\phi(b) = 0$ (You may assume that $\lambda > 0$.)
- (g) $\phi(0) = 0$ and $\frac{d\phi}{dx}(L) + \phi(L) = 0$ (If necessary, see Section 5.8.)