

**Exercise 2.3.3**

Consider the heat equation

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2},$$

subject to the boundary conditions

$$u(0, t) = 0 \quad \text{and} \quad u(L, t) = 0.$$

Solve the initial value problem if the temperature is initially

$$\begin{array}{ll} \text{(a)} & u(x, 0) = 6 \sin \frac{9\pi x}{L} \\ \text{(b)} & u(x, 0) = 3 \sin \frac{\pi x}{L} - \sin \frac{3\pi x}{L} \\ \text{(c)} & u(x, 0) = 2 \cos \frac{3\pi x}{L} \\ \text{(d)} & u(x, 0) = \begin{cases} 1 & 0 < x \leq L/2 \\ 2 & L/2 < x < L \end{cases} \\ \text{(e)} & u(x, 0) = f(x) \end{array}$$

[Your answer in part (c) may involve certain integrals that do not need to be evaluated.]