Exercise 2.3.7

Consider the following boundary value problem (if necessary, see Section 2.4.1):

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} \quad \text{with} \quad \frac{\partial u}{\partial x}(0,t) = 0, \ \frac{\partial u}{\partial x}(L,t) = 0, \quad \text{and} \quad u(x,0) = f(x).$$

(a) Give a one-sentence physical interpretation of this problem.

(b) Solve by the method of separation of variables. First show that there are no separated solutions which exponentially grow in time. [*Hint:* The answer is

$$u(x,t) = A_0 + \sum_{n=1}^{\infty} A_n e^{-\lambda_n k t} \cos \frac{n\pi x}{L}.\Bigg]$$

What is λ_n?

(c) Show that the initial condition, $u(x,0) = f(x)$, is satisfied if

$$f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos \frac{n\pi x}{L}.$$

(d) Using Exercise 2.3.6, solve for A_0 and A_n $(n \geq 1)$.

(e) What happens to the temperature distribution as $t \to \infty$? Show that it approaches the steady-state temperature distribution (see Section 1.4).