

Exercise 2.3.8

Consider

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} - \alpha u.$$

This corresponds to a one-dimensional rod either with heat loss through the lateral sides with outside temperature 0° ($\alpha > 0$, see Exercise 1.2.4) or with insulated lateral sides with a heat sink proportional to the temperature. Suppose that the boundary conditions are

$$u(0, t) = 0 \quad \text{and} \quad u(L, t) = 0.$$

- (a) What are the possible equilibrium temperature distributions if $\alpha > 0$?
- (b) Solve the time-dependent problem [$u(x, 0) = f(x)$] if $\alpha > 0$. Analyze the temperature for large time ($t \rightarrow \infty$) and compare to part (a).