Exercise 2.4.7

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}.$$

[*Hints*: It is known that if $u(x,t) = \phi(x)G(t)$, then $\frac{1}{kG}\frac{dG}{dt} = \frac{1}{\phi}\frac{d^2\phi}{dx^2}$. Appropriately assume $\lambda > 0$. Assume the eigenfunctions $\phi_n(x)$ satisfy the following integral condition (orthogonality):

$$\int_0^L \phi_n(x)\phi_m(x)\,dx = \begin{cases} 0 & n \neq m \\ L/2 & n = m \end{cases}$$

subject to the following conditions:

(a) $u(0,t) = 0$, $u(L,t) = 0$, $u(x,0) = f(x)$

(b) $u(0,t) = 0$, $\frac{\partial u}{\partial \mathrm{x}}(L,t) = 0$, $u(x,0) = f(x)$

(c) $\frac{\partial u}{\partial x}(0,t) = 0$, $u(L,t) = 0$, $u(x,0) = f(x)$

(d) $\frac{\partial u}{\partial \mathrm{x}}(0,t) = 0$, $\frac{\partial u}{\partial x}(L,t) = 0$, $u(x,0) = f(x)$ and modify orthogonal condition [using Table 2.4.1.]

[TYPO: These x's should be italicized. Also, the square bracket is not paired.]