

**Exercise 2.5.1**

Solve Laplace's equation inside a rectangle  $0 \leq x \leq L$ ,  $0 \leq y \leq H$ , with the following boundary conditions [*Hint*: Separate variables. If there are two homogeneous boundary conditions in  $y$ , let  $u(x, y) = h(x)\phi(y)$ , and if there are two homogeneous boundary conditions in  $x$ , let  $u(x, y) = \phi(x)h(y)$ .]:

$$(a) \quad \frac{\partial u}{\partial x}(0, y) = 0, \quad \frac{\partial u}{\partial x}(L, y) = 0, \quad u(x, 0) = 0, \quad u(x, H) = f(x)$$

$$(b) \quad \frac{\partial u}{\partial x}(0, y) = g(y), \quad \frac{\partial u}{\partial x}(L, y) = 0, \quad u(x, 0) = 0, \quad u(x, H) = 0$$

$$(c) \quad \frac{\partial u}{\partial x}(0, y) = 0, \quad u(L, y) = g(y), \quad u(x, 0) = 0, \quad u(x, H) = 0$$

$$(d) \quad u(0, y) = g(y), \quad u(L, y) = 0, \quad \frac{\partial u}{\partial y}(x, 0) = 0, \quad u(x, H) = 0$$

$$(e) \quad u(0, y) = 0, \quad u(L, y) = 0, \quad u(x, 0) - \frac{\partial u}{\partial y}(x, 0) = 0, \quad u(x, H) = f(x)$$

$$(f) \quad u(0, y) = f(y), \quad u(L, y) = 0, \quad \frac{\partial u}{\partial y}(x, 0) = 0, \quad \frac{\partial u}{\partial y}(x, H) = 0$$

$$(g) \quad \frac{\partial u}{\partial x}(0, y) = 0, \quad \frac{\partial u}{\partial x}(L, y) = 0, \quad u(x, 0) = \begin{cases} 0 & x > L/2 \\ 1 & x < L/2 \end{cases}, \quad \frac{\partial u}{\partial y}(x, H) = 0$$

$$(h) \quad u(0, y) = 0, \quad u(L, y) = g(y), \quad u(x, 0) = 0, \quad u(x, H) = 0$$