Exercise 2.5.2

Consider $u(x, y)$ satisfying Laplace's equation inside a rectangle ($0 < x < L$, $0 < y < H$) subject to the boundary conditions

$$\frac{\partial u}{\partial x}(0, y) = 0, \qquad \frac{\partial u}{\partial y}(x, 0) = 0$$
$$\frac{\partial u}{\partial x}(L, y) = 0, \qquad \frac{\partial u}{\partial y}(x, H) = f(x).$$

(a) *Without* solving this problem, briefly explain the physical condition under which there is a solution to this problem.

(b) Solve this problem by the method of separation of variables. Show that the method works only under the condition of part (a). [*Hint*: You may use (2.5.16) without derivation.]

(c) The solution [part (b)] has an arbitrary constant. Determine it by consideration of the time-dependent heat equation (1.5.11) subject to the initial condition

$$u(x, y, 0) = g(x, y).$$