Exercise 2.5.3

Solve Laplace's equation *outside* a circular disk ($r \geq a$) subject to the boundary condition [*Hint*: In polar coordinates,

$$\nabla^2 u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0,$$

it is known that if $u(r,\theta) = \phi(\theta)G(r)$, then $\frac{r}{G}\frac{d}{dr}\left(r\frac{dG}{dr}\right) = -\frac{1}{\phi}\frac{d^2\phi}{d\theta^2}$.]:

(a) $u(a,\theta) = \ln 2 + 4\cos 3\theta$

(b) $u(a,\theta) = f(\theta)$

You may assume that $u(r,\theta)$ remains finite as $r \to \infty$.