

**Exercise 2.5.4**

For Laplace's equation inside a circular disk ( $r \leq a$ ), using (2.5.45) and (2.5.47), show that

$$u(r, \theta) = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\bar{\theta}) \left[ -\frac{1}{2} + \sum_{n=0}^{\infty} \left(\frac{r}{a}\right)^n \cos n(\theta - \bar{\theta}) \right] d\bar{\theta}.$$

Using  $\cos z = \operatorname{Re}[e^{iz}]$ , sum the resulting geometric series to obtain Poisson's integral formula.