Problem 1.11

Volume of a parallelepiped
Show that the volume of a parallelepiped with edges **A**, **B**, and **C** is given by $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$.

Solution

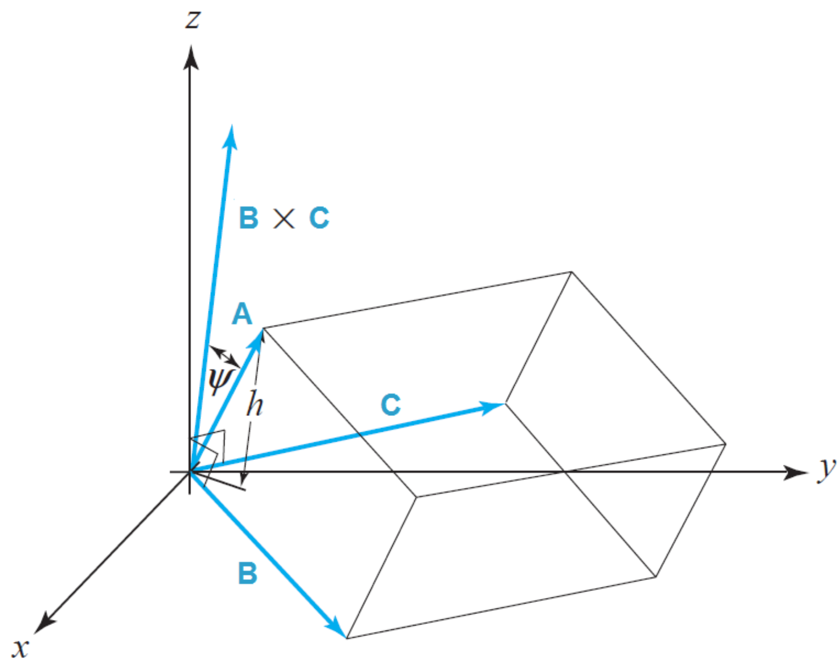

Figure 1: The three vectors, **A**, **B**, and **C**, are shown here.

The magnitude of the cross product of **B** and **C** gives the area of the parallelogram formed by **B** and **C**. In order to determine the volume of the parallelepiped, we have to multiply this area by the vertical height, h. h can be determined by considering the cosine of the angle, ψ, in the right triangle formed by h and **A**.

$$\cos\psi = \frac{h}{|\mathbf{A}|}$$

The vertical height is

$$h = |\mathbf{A}|\cos\psi,$$

so the volume is

$$V = |\mathbf{A}||\mathbf{B} \times \mathbf{C}|\cos\psi.$$

The product of the magnitudes and the cosine of the angle between the vectors is the definition of the dot product. Therefore, the volume of a parallelepiped with edges **A**, **B**, and **C** is given by

$$V = \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}).$$