Problem 1.22

Jerk
The rate of change of acceleration is known as "jerk." Find the direction and magnitude of jerk for a particle moving in a circle of radius R at angular velocity ω. Draw a vector diagram showing the instantaneous position, velocity, acceleration, and jerk.

Solution

A particle moving in a circle with radius R and angular velocity ω has the position vector,

$$\mathbf{r}(t) = \langle R\cos\omega t, R\sin\omega t\rangle.$$

Differentiate both sides with respect to t three times to obtain the jerk.

$$\begin{aligned}
\mathbf{r}'(t) &= \mathbf{v}(t) = \langle -R\omega\sin\omega t, R\omega\cos\omega t\rangle \\
\mathbf{r}''(t) &= \mathbf{a}(t) = \langle -R\omega^2\cos\omega t, -R\omega^2\sin\omega t\rangle \\
\mathbf{r}'''(t) &= \mathbf{j}(t) = \langle R\omega^3\sin\omega t, -R\omega^3\cos\omega t\rangle
\end{aligned}$$

The magnitude of jerk is

$$\begin{aligned}
|\mathbf{j}(\mathbf{t})| &= \sqrt{(R\omega^3)^2\sin^2\omega t + (-R\omega^3)^2\cos^2\omega t} \\
&= \sqrt{(R\omega^3)^2} \\
&= R\omega^3,
\end{aligned}$$

and the direction of jerk is

$$\begin{aligned}
\theta &= \tan^{-1}\left(\frac{-R\omega^3\cos\omega t}{R\omega^3\sin\omega t}\right) \\
&= \tan^{-1}(-\cot\omega t) \\
&= -\tan^{-1}\cot\omega t.
\end{aligned}$$

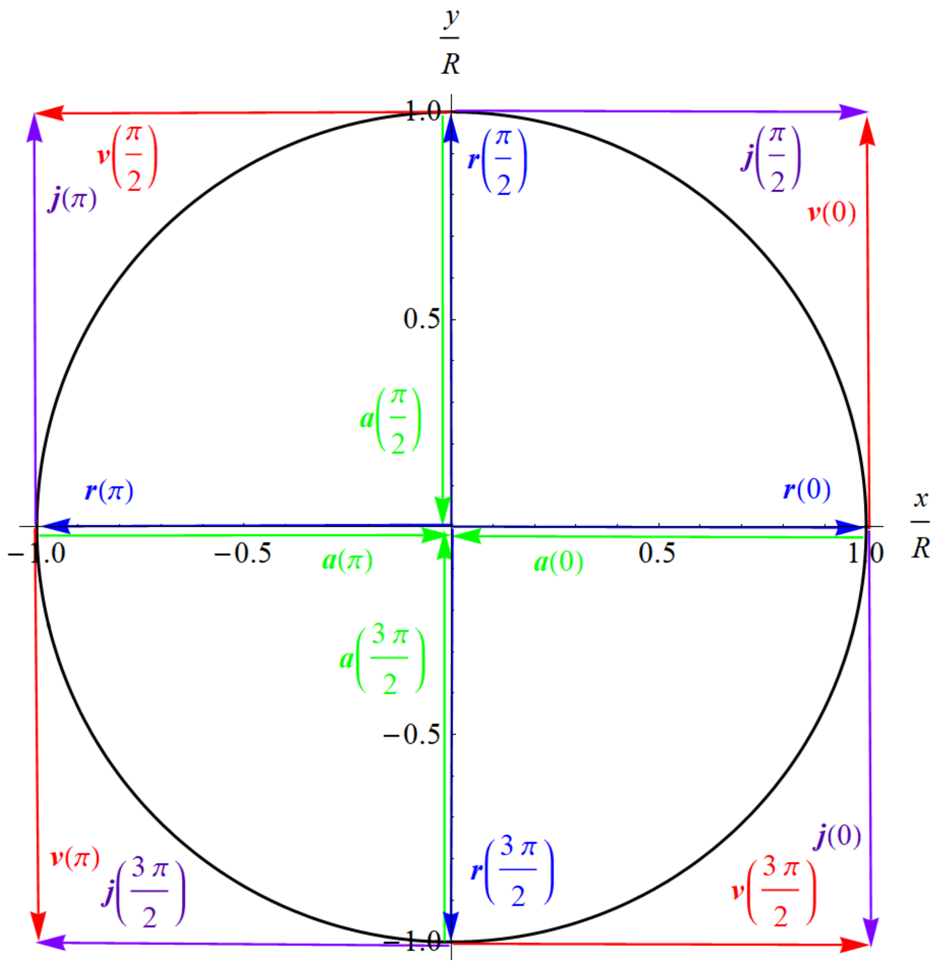

Figure 1: This figure illustrates the position (blue), velocity (red), acceleration (green), and jerk (purple) vectors for $t = 0$, $t = \pi/2$, $t = \pi$, and $t = 3\pi/2$ for the special value of $\omega = 1$. In this case the vectors all have the same magnitude R (generally they are different).