

Exercise 16

In Exercises 13 to 19, use set theoretic or vector notation or both to describe the points that lie in the given configurations.

The line passing through $(0, 2, 1)$ in the direction of $2\mathbf{i} - \mathbf{k}$

Solution

The equation for a line is

$$\mathbf{y}(t) = \mathbf{m}t + \mathbf{b},$$

where \mathbf{m} is the direction vector, \mathbf{b} is the position vector for a point on the line, and t is a parameter. The vector $2\mathbf{i} - \mathbf{k}$ can be written as $(2, 0, -1)$, so the line in question can be written as

$$\begin{aligned}\mathbf{y}(t) &= (2, 0, -1)t + (0, 2, 1) \\ &= (2t, 0, -t) + (0, 2, 1) \\ &= (2t, 2, -t + 1)\end{aligned}$$

Since there's only one arbitrary constant t , the line is one-dimensional. The set of points on the line is described by

$$\{(2t, 2, -t + 1), t \in \mathbb{R}\}.$$