Exercise 17

In Exercises 13 to 19, use set theoretic or vector notation or both to describe the points that lie in the given configurations.

The line passing through $(-1,-1,-1)$ and $(1,-1,2)$

Solution

The equation for a line is

$$\mathbf{y}(t) = \mathbf{m}t + \mathbf{b},$$

where $\mathbf{m}$ is the direction vector, b is the position vector for a point on the line, and t is a parameter. Subtract the two given position vectors to get $\mathbf{m}$.

$$\begin{aligned}\mathbf{m} &= (1,-1,2) - (-1,-1,-1) \\ &= (2,0,3)\end{aligned}$$

So the line in question can be written as

$$\begin{aligned}\mathbf{y}(t) &= (2,0,3)t + (-1,-1,-1) \\ &= (2t,0,3t) + (-1,-1,-1) \\ &= (2t-1,-1,3t-1)\end{aligned}$$

Since there's only one arbitrary constant t, the line is one-dimensional. The set of points on the line is described by

$$\{(2t-1,-1,3t-1),\ t \in \mathbb{R}\}.$$