Exercise 25

Show that there are no points (x, y, z) satisfying $2x - 3y + z - 2 = 0$ and lying on the line $\mathbf{v} = (2, -2, -1) + t(1, 1, 1)$.

Solution

The parameterization for the line can be written as

$$\begin{aligned}\mathbf{v} &= (2, -2, -1) + t(1, 1, 1) \\ &= (2, -2, -1) + (t, t, t) \\ &= (2 + t, -2 + t, -1 + t).\end{aligned}$$

The x-, y-, and z-components of the line are

$$x = 2 + t \quad \text{and} \quad y = -2 + t \quad \text{and} \quad z = -1 + t,$$

respectively. Substitute these into the equation for the plane.

$$\begin{aligned}2x - 3y + z - 2 &= 2(2 + t) - 3(-2 + t) + (-1 + t) - 2 \\ &= 4 + 2t + 6 - 3t - 1 + t - 2 \\ &= 7 \neq 0\end{aligned}$$

Since the right side is 7 for all values of t and not 0, there are no points (x, y, z) on the line that lie in the plane.