Exercise 20

Find the projection of $\mathbf{u} = -\mathbf{i} + \mathbf{j} + \mathbf{k}$ onto $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$.

Solution

The dot product of $\mathbf{u}$ and $\hat{\mathbf{v}}$ represents the component of $\mathbf{u}$ in the direction of $\mathbf{v}$.

$$\begin{aligned}\mathbf{u} \cdot \hat{\mathbf{v}} &= \mathbf{u} \cdot \frac{\mathbf{v}}{\|\mathbf{v}\|} \\ &= \frac{(-\mathbf{i} + \mathbf{j} + \mathbf{k}) \cdot (2\mathbf{i} + \mathbf{j} - 3\mathbf{k})}{\sqrt{2^2 + 1^2 + (-3)^2}} \\ &= \frac{(-1)(2) + (1)(1) + (1)(-3)}{\sqrt{14}} \\ &= -\frac{4}{\sqrt{14}}\end{aligned}$$

Multiply this result by a unit vector in the direction of $\mathbf{v}$ to obtain the desired projection.

$$\begin{aligned}(\mathbf{u} \cdot \hat{\mathbf{v}})\hat{\mathbf{v}} &= (\mathbf{u} \cdot \hat{\mathbf{v}})\frac{\mathbf{v}}{\|\mathbf{v}\|} \\ &= \left(-\frac{4}{\sqrt{14}}\right)\frac{2\mathbf{i} + \mathbf{j} - 3\mathbf{k}}{\sqrt{2^2 + 1^2 + (-3)^2}} \\ &= \left(-\frac{4}{\sqrt{14}}\right)\frac{2\mathbf{i} + \mathbf{j} - 3\mathbf{k}}{\sqrt{14}} \\ &= -\frac{2}{7}(2\mathbf{i} + \mathbf{j} - 3\mathbf{k})\end{aligned}$$