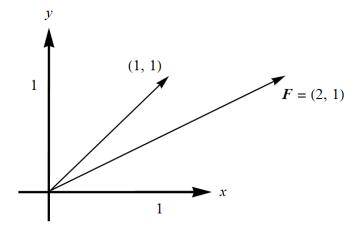
## Exercise 36

Suppose that an object moving in direction  $\mathbf{i} + \mathbf{j}$  is acted on by a force given by the vector  $2\mathbf{i} + \mathbf{j}$ . Express this force as the sum of a force in the direction of motion and a force perpendicular to the direction of motion.

## Solution



The aim is to decompose the force vector  $\mathbf{F}$  into components parallel and perpendicular to the direction of motion.

$$\mathbf{F} = \mathbf{F}_{\parallel} + \mathbf{F}_{\perp}$$

 $\mathbf{F}_{\parallel}$  will be found first. Start by taking the dot product of  $\mathbf{F}$  with a unit vector in the direction of motion.

$$(2,1) \cdot \frac{(1,1)}{\sqrt{1^2 + 1^2}} = \frac{(2,1) \cdot (1,1)}{\sqrt{2}} = \frac{3}{\sqrt{2}}$$

This represents the component of  $\mathbf{F}$  parallel to the motion. Multiply it by the unit vector in this direction to get  $\mathbf{F}_{\parallel}$ .

$$\mathbf{F}_{\parallel} = \frac{3}{\sqrt{2}} \frac{(1,1)}{\sqrt{1^2 + 1^2}} = \frac{3}{2} (1,1)$$

To get  $\mathbf{F}_{\perp}$ , subtract  $\mathbf{F}_{\parallel}$  from  $\mathbf{F}$ .

$$\begin{aligned} \mathbf{F}_{\perp} &= \mathbf{F} - \mathbf{F}_{\parallel} \\ &= (2,1) - \frac{3}{2}(1,1) \\ &= (2,1) - \left(\frac{3}{2}, \frac{3}{2}\right) \\ &= \left(\frac{1}{2}, -\frac{1}{2}\right) \\ &= \frac{1}{2}(1,-1) \end{aligned}$$

Therefore,

$$\mathbf{F} = \frac{3}{2}(1,1) + \frac{1}{2}(1,-1).$$