Exercise 13

Compute $\mathbf{u}+\mathbf{v}$, $\mathbf{u}\cdot\mathbf{v}$, $\|\mathbf{u}\|$, $\|\mathbf{v}\|$, and $\mathbf{u}\times\mathbf{v}$, where $\mathbf{u}=\mathbf{i}-2\mathbf{j}+\mathbf{k}$, $\mathbf{v}=2\mathbf{i}-\mathbf{j}+2\mathbf{k}$.

Solution

Write each of the vectors as

$$\begin{aligned}\mathbf{u} &= (1,-2,1)\\ \mathbf{v} &= (2,-1,2).\end{aligned}$$

Then

$$\begin{aligned}\|\mathbf{u}\| &= \sqrt{1^2+(-2)^2+1^2} = \sqrt{6}\\ \|\mathbf{v}\| &= \sqrt{2^2+(-1)^2+2^2} = \sqrt{9} = 3\\ \mathbf{u}+\mathbf{v} &= (1,-2,1)+(2,-1,2) = (3,-3,3)\\ \mathbf{u}\cdot\mathbf{v} &= (1,-2,1)\cdot(2,-1,2) = (1)(2)+(-2)(-1)+(1)(2) = 6\\ \mathbf{u}\times\mathbf{v} &= \begin{vmatrix}\hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}}\\ 1 & -2 & 1\\ 2 & -1 & 2\end{vmatrix} = (-4+1)\hat{\mathbf{x}} - (2-2)\hat{\mathbf{y}} + (-1+4)\hat{\mathbf{z}} = -3\hat{\mathbf{x}} + 3\hat{\mathbf{z}} = (-3,0,3).\end{aligned}$$