

Exercise 2

Evaluate the determinants

$$(a) \begin{vmatrix} 2 & -1 & 0 \\ 4 & 3 & 2 \\ 3 & 0 & 1 \end{vmatrix} \qquad (b) \begin{vmatrix} 36 & 18 & 17 \\ 45 & 24 & 20 \\ 3 & 5 & -2 \end{vmatrix}$$

$$(c) \begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix} \qquad (d) \begin{vmatrix} 2 & 3 & 5 \\ 7 & 11 & 13 \\ 17 & 19 & 23 \end{vmatrix}$$

Solution

$$\begin{aligned} \begin{vmatrix} 2 & -1 & 0 \\ 4 & 3 & 2 \\ 3 & 0 & 1 \end{vmatrix} &= 3 \begin{vmatrix} -1 & 0 \\ 3 & 2 \end{vmatrix} + 1 \begin{vmatrix} 2 & -1 \\ 4 & 3 \end{vmatrix} \\ &= 3[(-1)(2) - (0)(3)] + 1[(2)(3) - (-1)(4)] \\ &= 4 \end{aligned}$$

$$\begin{aligned} \begin{vmatrix} 36 & 18 & 17 \\ 45 & 24 & 20 \\ 3 & 5 & -2 \end{vmatrix} &= 3 \begin{vmatrix} 18 & 17 \\ 24 & 20 \end{vmatrix} - 5 \begin{vmatrix} 36 & 17 \\ 45 & 20 \end{vmatrix} + (-2) \begin{vmatrix} 36 & 18 \\ 45 & 24 \end{vmatrix} \\ &= 3[(18)(20) - (17)(24)] - 5[(36)(20) - (17)(45)] + (-2)[(36)(24) - (18)(45)] \\ &= -27 \end{aligned}$$

$$\begin{aligned} \begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix} &= 1 \begin{vmatrix} 9 & 16 \\ 16 & 25 \end{vmatrix} - 4 \begin{vmatrix} 4 & 16 \\ 9 & 25 \end{vmatrix} + 9 \begin{vmatrix} 4 & 9 \\ 9 & 16 \end{vmatrix} \\ &= 1[(9)(25) - (16)(16)] - 4[(4)(25) - (16)(9)] + 9[(4)(16) - (9)(9)] \\ &= -8 \end{aligned}$$

$$\begin{aligned} \begin{vmatrix} 2 & 3 & 5 \\ 7 & 11 & 13 \\ 17 & 19 & 23 \end{vmatrix} &= 2 \begin{vmatrix} 11 & 13 \\ 19 & 23 \end{vmatrix} - 3 \begin{vmatrix} 7 & 13 \\ 17 & 23 \end{vmatrix} + 5 \begin{vmatrix} 7 & 11 \\ 17 & 19 \end{vmatrix} \\ &= 2[(11)(23) - (13)(19)] - 3[(7)(23) - (13)(17)] + 5[(7)(19) - (11)(17)] \\ &= -78 \end{aligned}$$