

Exercise 27

Let $\mathbf{v} = (1, 1, 0)$ and $\mathbf{w} = (0, 2, -1)$. Use the algebraic rules and multiplication table on page 37 to compute $\mathbf{v} \times \mathbf{w}$ without using determinants.

Solution

$$\begin{aligned}\mathbf{v} \times \mathbf{w} &= (1, 1, 0) \times (0, 2, -1) \\ &= (\hat{\mathbf{x}} + \hat{\mathbf{y}}) \times (2\hat{\mathbf{y}} - \hat{\mathbf{z}}) \\ &= \hat{\mathbf{x}} \times (2\hat{\mathbf{y}} - \hat{\mathbf{z}}) + \hat{\mathbf{y}} \times (2\hat{\mathbf{y}} - \hat{\mathbf{z}}) \\ &= 2(\hat{\mathbf{x}} \times \hat{\mathbf{y}}) - (\hat{\mathbf{x}} \times \hat{\mathbf{z}}) + 2(\hat{\mathbf{y}} \times \hat{\mathbf{y}}) - (\hat{\mathbf{y}} \times \hat{\mathbf{z}}) \\ &= 2(\hat{\mathbf{z}}) - (-\hat{\mathbf{y}}) + 2(\mathbf{0}) - (\hat{\mathbf{x}}) \\ &= -\hat{\mathbf{x}} + \hat{\mathbf{y}} + 2\hat{\mathbf{z}} \\ &= (-1, 1, 2)\end{aligned}$$