Exercise 29

Find an equation for the plane that passes through the point $(1, 2, -3)$ and is perpendicular to the line $\mathbf{v} = (0, -2, 1) + t(1, -2, 3)$.

Solution

The equation for a plane is

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0,$$

where $\mathbf{n}$ is a vector normal to the plane and $\mathbf{r}_0$ is the position vector for any point in the plane. The direction vector of the line, $(1, -2, 3)$, serves as a normal vector to the plane, and $(1, 2, -3)$ is the needed position vector.

$$(1, -2, 3) \cdot (x - 1, y - 2, z + 3) = 0$$

$$1(x - 1) - 2(y - 2) + 3(z + 3) = 0$$

$$x - 1 - 2y + 4 + 3z + 9 = 0$$

$$x - 2y + 3z = -12$$