Exercise 34

Find the distance from the point $(2, 1, -1)$ to the plane $x - 2y + 2z + 5 = 0$.

Solution

The normal vector to the plane $\mathbf{n}$ is obtained from the coefficients of x, y, and z: $\mathbf{n} = (1, -2, 2)$. An equation for the line with direction vector $(1, -2, 2)$ that passes through $(2, 1, -1)$ is

$$\begin{aligned} \mathbf{y}(t) &= (1, -2, 2)t + (2, 1, -1) \\ &= (t, -2t, 2t) + (2, 1, -1) \\ &= (t + 2, -2t + 1, 2t - 1). \end{aligned}$$

Substitute $x = t + 2$, $y = -2t + 1$, and $z = 2t - 1$ into the equation for the plane and solve for t to find when the line intersects the plane.

$$(t + 2) - 2(-2t + 1) + 2(2t - 1) + 5 = 0 \quad \to \quad t = -\frac{1}{3}$$

The point at which the line intersects the plane is then

$$\mathbf{y}\left(-\frac{1}{3}\right) = \left(-\frac{1}{3} + 2, -2\frac{-1}{3} + 1, 2\frac{-1}{3} - 1\right) = \left(\frac{5}{3}, \frac{5}{3}, -\frac{5}{3}\right).$$

Therefore, the perpendicular distance from $(2, 1, -1)$ to the plane is

$$d = \sqrt{\left(2 - \frac{5}{3}\right)^2 + \left(1 - \frac{5}{3}\right)^2 + \left(-1 + \frac{5}{3}\right)^2} = 1.$$