

Exercise 44

Justify the steps in the following computation:

$$\begin{aligned} \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{vmatrix} &= \begin{vmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 7 & 8 & 10 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -11 \end{vmatrix} \\ &= \begin{vmatrix} -3 & -6 \\ -6 & -11 \end{vmatrix} = 33 - 36 = -3. \end{aligned}$$

Solution

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{vmatrix}$$

Multiply the first row by -4 and add the respective elements to the second row.

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 + (-4)1 & 5 + (-4)2 & 6 + (-4)3 \\ 7 & 8 & 10 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 7 & 8 & 10 \end{vmatrix}$$

Multiply the first row by -7 and add the respective elements to the third row.

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 7 + (-7)1 & 8 + (-7)2 & 10 + (-7)3 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -11 \end{vmatrix}$$

Use expansion by minors along the first column to evaluate the determinant.

$$\begin{aligned} \begin{vmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -11 \end{vmatrix} &= 1 \begin{vmatrix} -3 & -6 \\ -6 & -11 \end{vmatrix} - 0 \begin{vmatrix} 2 & 3 \\ -6 & -11 \end{vmatrix} + 0 \begin{vmatrix} 2 & 3 \\ -3 & -6 \end{vmatrix} \\ &= \begin{vmatrix} -3 & -6 \\ -6 & -11 \end{vmatrix} \\ &= (-3)(-11) - (-6)(-6) \\ &= 33 - 36 \\ &= -3 \end{aligned}$$