

Problem 7

- (a) Show that for
- $xy \neq 1$
- ,

$$\arctan x - \arctan y = \arctan \frac{x - y}{1 + xy}$$

if the left side lies between $-\pi/2$ and $\pi/2$.

- (b) Show that
- $\arctan \frac{120}{119} - \arctan \frac{1}{239} = \pi/4$
- .

- (c) Deduce the following formula of John Machin (1680-1751):

$$4 \arctan \frac{1}{5} - \arctan \frac{1}{239} = \frac{\pi}{4}$$

- (d) Use the Maclaurin series for
- \arctan
- to show that

$$0.1973955597 < \arctan \frac{1}{5} < 0.1973955616$$

- (e) Show that

$$0.004184075 < \arctan \frac{1}{239} < 0.004184077$$

- (f) Deduce that, correct to seven decimal places,
- $\pi \approx 3.1415927$
- .

Machin used this method in 1706 to find π correct to 100 decimal places. Recently, with the aid of computers, the value of π has been computed to increasingly greater accuracy. In 2013 Shigeru Kondo and Alexander Yee computed the value of π to more than 12 trillion decimal places!