

**Problem 3**

Let  $f$  be a function with the property that  $f(0) = 1$ ,  $f'(0) = 1$ , and  $f(a + b) = f(a)f(b)$  for all real numbers  $a$  and  $b$ . Show that  $f'(x) = f(x)$  for all  $x$  and deduce that  $f(x) = e^x$ .