

Problem 9

A dog sees a rabbit running in a straight line across an open field and gives chase. In a rectangular coordinate system (as shown in the figure), assume:

- (i) The rabbit is at the origin and the dog is at the point $(L, 0)$ at the instant the dog first sees the rabbit.
- (ii) The rabbit runs up the y -axis and the dog always runs straight for the rabbit.
- (iii) The dog runs at the same speed as the rabbit.

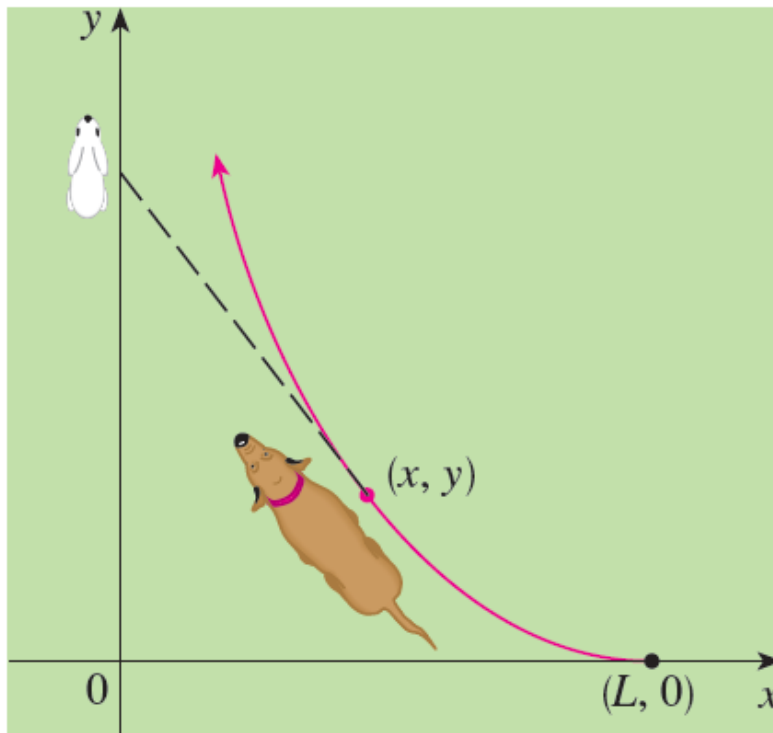


FIGURE FOR PROBLEM 9

- (a) Show that the dog's path is the graph of the function $y = f(x)$, where y satisfies the differential equation

$$x \frac{d^2 y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx} \right)^2}$$

- (b) Determine the solution of the equation in part (a) that satisfies the initial conditions $y = y' = 0$ when $x = L$. [Hint: Let $z = dy/dx$ in the differential equation and solve the resulting first-order equation to find z ; then integrate z to find y .]
- (c) Does the dog ever catch the rabbit?