# Exercise 29

Find the orthogonal trajectories of the family of curves. Use a graphing device to draw several members of each family on a common screen.

$$x^2 + 2y^2 = k^2$$

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**Solution**

To find the orthogonal trajectories, we have to solve for $y'(x)$, set $y'_{\perp}$ equal to the negative reciprocal, and then solve for $y_{\perp}$. Start by differentiating both sides of the given equation with respect to $x$.

$$\frac{d}{dx}\left(x^2 + 2y^2\right) = \frac{d}{dx}\left(k^2\right)$$
$$2x + 2 * 2y\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = -\frac{x}{2y}$$

Here is where we introduce $y_{\perp}$.

$$\frac{dy_{\perp}}{dx} = \frac{2y_{\perp}}{x}$$

Since this equation is separable, we can solve for $y_{\perp}$ by bringing all terms with $y_{\perp}$ to the left and all constants and terms with $x$ to the right and then integrating both sides.

$$dy_{\perp} = \frac{2y_{\perp}}{x}\,dx$$
$$\frac{dy_{\perp}}{y_{\perp}} = \frac{2}{x}\,dx$$
$$\ln|y_{\perp}| = 2\ln|x| + C$$

Exponentiate both sides.

$$e^{\ln|y_{\perp}|} = e^{2\ln|x|+C}$$
$$|y_{\perp}| = |x|^2 e^C$$
$$y_{\perp} = \pm|x|^2 e^C$$
$$y_{\perp} = Ax^2$$

Therefore, the orthogonal trajectories are the family of parabolas centered at the origin.

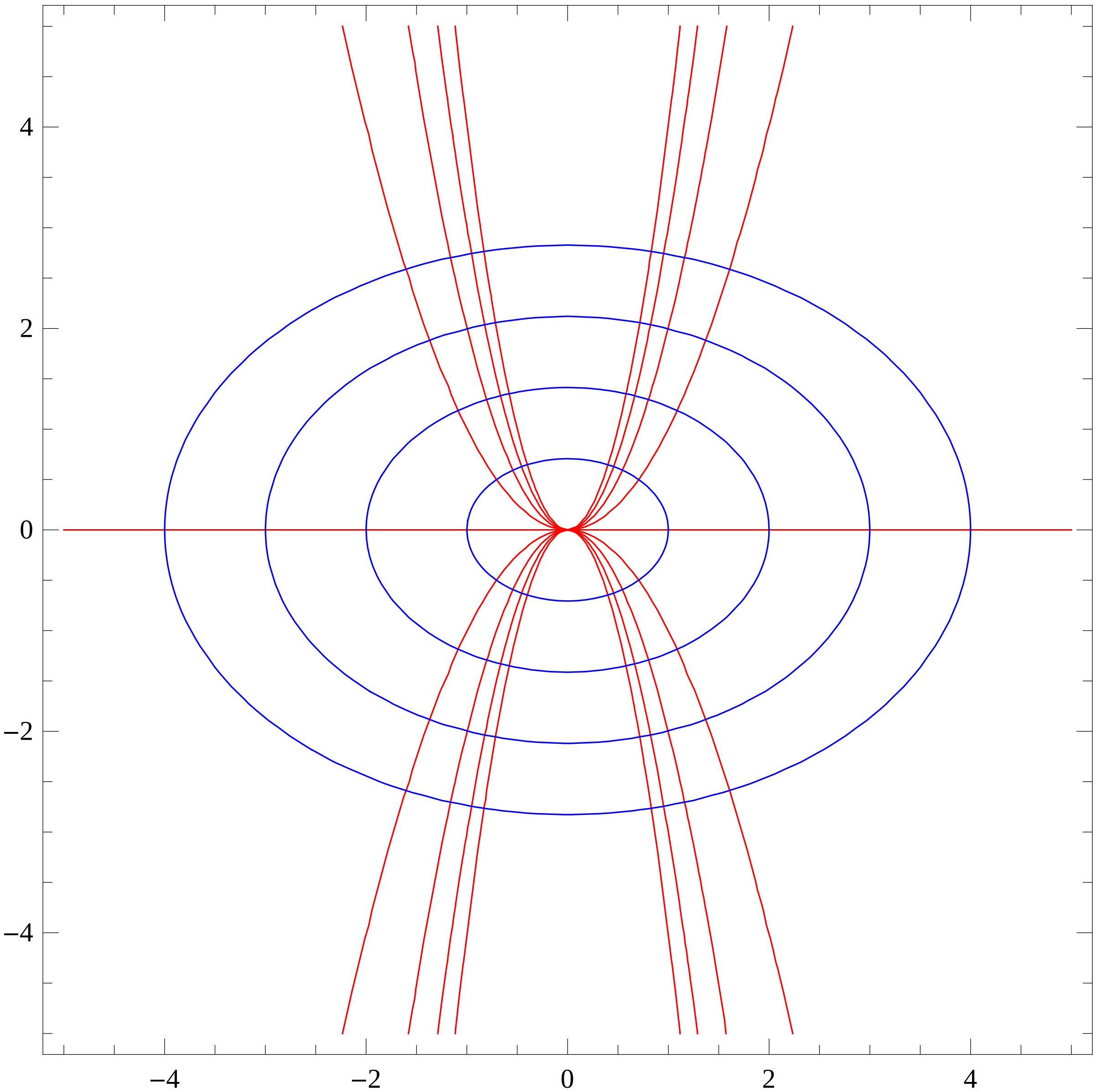


Figure 1: Plot of the ellipses ($k = 1, 2, 3, 4$) and their orthogonal trajectories ($A = -4, -3, \ldots, 3, 4$).