**Exercise 6**

Are the three vectors $[1,2,3]$, $[-2,0,1]$, and $[1,10,17]$ linearly dependent or independent? Do they span all vectors or not?

**Solution**

By definition, a set of vectors $S = \{v_1, v_2, \ldots, v_n\}$ is said to be linearly independent when the only solution to the homogeneous equation

$$\alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n = 0$$

is the trivial solution $\alpha_1 = \alpha_2 = \cdots = \alpha_n = 0$. Otherwise, the set is linearly dependent.

For this particular exercise, $v_1 = [1, 2, 3]$ and $v_2 = [-2, 0, 1]$ and $v_3 = [1, 10, 17]$. There is a nontrivial solution:

$$5v_1 + 2v_2 + (-1)v_3 = 0$$

Therefore, the three vectors are linearly dependent. A more systematic way of coming to this conclusion is the following: Arrange the vectors as the columns (or rows) of a matrix and find the determinant of this matrix. If the determinant is nonzero, then the vectors are linearly independent; otherwise, they are linearly dependent.

$$\begin{vmatrix} 1 & 2 & 3 \\ -2 & 0 & 1 \\ 1 & 10 & 17 \end{vmatrix} = 1 \begin{vmatrix} 0 & 1 \\ 10 & 17 \end{vmatrix} - 2 \begin{vmatrix} -2 & 1 \\ 1 & 17 \end{vmatrix} + 3 \begin{vmatrix} -2 & 0 \\ 1 & 10 \end{vmatrix}$$

$$= 1(-10) - 2(-34 - 1) + 3(-20)$$

$$= 0$$

This confirms the result. Because $v_3$ can be written in terms of $v_1$ and $v_2$, the vectors only span a plane in $\mathbb{R}^3$, not all of $\mathbb{R}^3$.