

Exercise 12

Show that the new coordinate axes defined by (3) are orthogonal.

Solution

The coordinate axes defined by (3) are

$$x' = ax + by \quad y' = bx - ay. \quad (3)$$

Treat these coordinate axes as vectors in the plane. We can tell if they are orthogonal by seeing if the dot product of the two is 0. Recall that the dot product is the sum of the product of the x -components and the product of the y -components.

$$\begin{aligned} x' &= [a, b] \\ y' &= [b, -a] \end{aligned}$$

$$x' \cdot y' = ab + b(-a) = ab - ab = 0$$

Because the dot product is 0, the coordinate axes defined by (3) are orthogonal.