

Exercise 9

This is an exercise on the divergence theorem

$$\iiint_D \nabla \cdot \mathbf{F} \, d\mathbf{x} = \iint_{\text{bdy } D} \mathbf{F} \cdot \mathbf{n} \, dS,$$

valid for any bounded domain D in space with boundary surface $\text{bdy } D$ and unit outward normal vector \mathbf{n} . If you never learned it, see Section A.3. It is crucial that D be bounded. As an exercise, verify it in the following case by calculating both sides separately: $\mathbf{F} = r^2\mathbf{x}$, $\mathbf{x} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, $r^2 = x^2 + y^2 + z^2$, and $D =$ the ball of radius a and center at the origin.