

Exercise 1

By trial and error, find a solution of the diffusion equation $u_t = u_{xx}$ with the initial condition $u(x, 0) = x^2$.

Solution

If we take two derivatives of the initial condition with respect to x , we get

$$x^2 \rightarrow 2x \rightarrow 2.$$

What function of time can we differentiate once to get 2? The answer is $2t + C$. But because we want this function to disappear when we plug in $t = 0$, we ignore the integration constant. Therefore, the solution of the diffusion equation is

$$u(x, t) = x^2 + 2t.$$