

## Exercise 4

Consider the Neumann problem

$$\begin{aligned}\Delta u &= f(x, y, z) \quad \text{in } D \\ \frac{\partial u}{\partial n} &= 0 \quad \text{on bdy } D.\end{aligned}$$

- (a) What can we surely add to any solution to get another solution? So we don't have uniqueness.
- (b) Use the divergence theorem and the PDE to show that

$$\iiint_D f(x, y, z) \, dx \, dy \, dz = 0$$

is a necessary condition for the Neumann problem to have a solution.

- (c) Can you give a physical interpretation of part (a) and/or (b) for either heat flow or diffusion?