

Exercise 4

Consider the eigenvalue problem $-\Delta v = \lambda v$ in the unit square $D = \{0 < x < 1, 0 < y < 1\}$ with the Dirichlet BC $v = 0$ on the bottom and both vertical sides, and the Robin BC $\partial v / \partial y = -v$ on the top $\{y = 1\}$.

- (a) Show that all the eigenvalues are positive.
- (b) Find an equation for the eigenvalues λ . Show that they can be expressed in terms of the roots of the equation $s + \tan s = 0$.
- (c) Find the solutions of the last equation graphically. Find an approximate formula for the (m, n) th eigenvalue for large (m, n) .