Exercise 5

Find the dimension of each of the following vector spaces.

(a) The space of all the solutions of $u'' + x^2 u = 0$.

(b) The eigenspace with eigenvalue $(2\pi/l)^2$ of the operator $-d^2/dt^2$ on the interval $(-l, l)$ with the periodic boundary conditions.

(c) The space of harmonic functions in the unit disk with the homogeneous Neumann BCs.

(d) The eigenspace with eigenvalue $\lambda = 25\pi^2$ of $-\Delta$ in the unit square $(0, 1)^2$ with the homogeneous Neumann BCs on all four sides.

(e) The space of all the solutions of $u_{tt} = c^2 u_{xx}$ in $-\infty < x < \infty$, $-\infty < t < \infty$.