Exercise 3

Suppose that you have a circular drum with wave speed $c_d$ and radius $a$ and a violin string with wave speed $c_v$ and length $l$. In order to make the fundamental frequencies of the drum and the violin the same, how would you choose the length $l$?

Solution

The fundamental frequency of a circular drum is

$$\nu_d = \frac{\alpha_{01} c_d}{a},$$

where $\alpha_{01} \approx 2.40483$ is the first positive zero of $J_0$, the zero-order Bessel function of the first kind.

The fundamental frequency of a violin string is

$$\nu_v = \frac{\pi c_v}{l},$$

where $\pi \approx 3.14159$ is the first positive zero of sine. Set the two frequencies to be equal and solve for $l$.

$$\frac{\pi c_v}{l} = \frac{\alpha_{01} c_d}{a}$$

Therefore,

$$l = \frac{\pi ac_v}{\alpha_{01} c_d}.$$