

Exercise 5

(*The hammer blow*) Let $\phi(x) \equiv 0$ and $\psi(x) = 1$ for $|x| < a$ and $\psi(x) = 0$ for $|x| \geq a$. Sketch the string profile (u versus x) at each of the successive instants $t = a/2c, a/c, 3a/2c, 2a/c,$ and $5a/c$. [Hint: Calculate

$$u(x, t) = \frac{1}{2c} \int_{x-ct}^{x+ct} \psi(s) ds = \frac{1}{2c} \{\text{length of } (x - ct, x + ct) \cap (-a, a)\}.$$

Then $u(x, a/2c) = (1/2c) \{\text{length of } (x - a/2, x + a/2) \cap (-a, a)\}$. This takes on different values for $|x| < a/2$, for $a/2 < x < 3a/2$, and for $x > 3a/2$. Continue in this manner for each case.]