

## Exercise 8

A *spherical wave* is a solution of the three-dimensional wave equation of the form  $u(r, t)$ , where  $r$  is the distance to the origin (the spherical coordinate). The wave equation takes the form

$$u_{tt} = c^2 \left( u_{rr} + \frac{2}{r} u_r \right) \quad (\text{“spherical wave equation”}).$$

- (a) Change variables  $v = ru$  to get the equation for  $v$ :  $v_{tt} = c^2 v_{rr}$ .
- (b) Solve for  $v$  using (3) and thereby solve the spherical wave equation.
- (c) Use (8) to solve it with initial conditions  $u(r, 0) = \phi(r)$ ,  $u_t(r, 0) = \psi(r)$ , taking both  $\phi(r)$  and  $\psi(r)$  to be even functions of  $r$ .